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AN INVERSE PROBLEM IN THE THEORY OF TWO-PHASE FILTRATION†

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Algorithms for solving the inverse problem of determining the relative phase permeability (RPP) functions data from unsteady laboratory investigations of samples of a porous medium are considered. It is proposed that "a priori" information on the form of the RPPs, obtained during the course of steady-state investigations of lithologically similar samples of the porous medium, should be used to regularize this problem. Methods are developed for determining the optimal complexity of the relations which approximate the RPPs. Examples of the use of the proposed algorithms to solve a model problem and for processing real data are given. Copyright © 1996 Elsevier Science Ltd.

1. FORMULATION OF THE PROBLEM

The displacements of a fluid from a sample of a porous medium is described by the Buckley–Leverett equation

$$\frac{ds}{d\tau} + f'(s)\frac{ds}{d\xi} = 0, \quad f(s) = \frac{f_1(s)}{f_1(s) + \mu_0 f_2(s)}, \quad \xi = \frac{x}{l} \tag{1.1}$$

where s is the saturation by the displacing agent, τ is the dimensionless time, which is equal to the ratio of the volume of the displacing agent to the overall pore volume, x is a spatial coordinate, l is the sample length, $f(s)$ is the derivative of the Buckley–Leverett function $f(s)$, $f_1(s)$ and $f_2(s)$ are the relative phase permeabilities (RPPs) of the displacing agent and the fluid, respectively and $\mu_0 = \mu_1/\mu_2$ is the ratio of their viscosities.

The solution of Eq. (1.1) which satisfies the conditions $s(0, \tau) = s_T$, $s(\xi, 0) = s_c$ has the form [1]

$$s = \begin{cases} \Psi(\xi/\tau), & \xi < \xi_c(\tau) \\ s_c, & \xi \geqslant \xi_c(\tau) \end{cases}$$

where s_c and s_T are the initial and final saturation of the porous medium by the displacing agent, Ψ is a function which is the result of the inversion of the function $f'(s)$ in $[s_*, s_T]$, $\xi_c = v\tau$, $v = f(s_*)$ is the velocity of motion of the front and s_* is the saturation value at the displacement front, which is determined from the condition

$$f'(s_*) = (f(s_*) - f(s_C))/(s_* - s_C)$$

The dependence of the dimensionless pressure drop $\Delta P(\tau)$ on time is given by the expression

$$\Delta P(\tau) = \int_0^1 \frac{d\xi}{\Phi(s(\xi,\tau))}, \quad \tau \geqslant \tau_*; \quad \Delta P(\tau) = \int_0^{\xi_c} \frac{d\xi}{\Phi(s(\xi,\tau))} + \frac{1-\xi_c}{\mu_0 F_2}, \quad \tau < \tau_*$$

where $\tau_* = 1/f(s_*)$ is the dimensionless time after which the displacement front reaches the outlet from the sample ($\xi = 1$) and F_2 is the RPP of the displaced fluid when $s = s_c$. On changing to dimensionless variables, the quantity $\Delta P_m = \Delta P k/(\mu_1 l v)$, where k is the permeability with respect to air, l is the sample length, v is the filtration velocity and μ_1 is the viscosity of the displacing agent, is adopted as the pressure scale. The volume of the displaced fluid, divided by the pore volume, is represented as

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$$V_2(\tau) = \tau, \quad \tau < \tau_*; \quad V_2(\tau) = \tau_* + \int_{\tau_*}^{\tau} [1 - f(s(1,t))]dt, \quad \tau \geqslant \tau_*$$

The inverse problem of determining the RPPs $f_1(s)$ and $f_2(s)$ is formulated using measurements of the pressure drop $\Delta P^0(\tau_i)$ and the volume of the displaced fluid $V_2^0(\tau_i)$ (τ_i is the time of the ith measurement, $i = 1, \ldots, M$ and M is the sample volume).

It should be noted that the RPPs are determined most accurately using data obtained from steady-state investigations during the course of which the displacing agent and the displaced fluid penetrate into a sample of the porous medium in a definite proportion and, for each set of conditions, time is allowed for steady-state filtration to be established (that is, for the readings of the instruments which measure the pressure gradient and the saturation by water of the sample of the porous medium to stabilize). However, these measurements are very time consuming, which limits the number of samples which can be investigated. An estimate of the RPPs is therefore often carried out using data from unsteady-state investigations in accordance with the formulation considered above. However, the implementation of this method is complicated by the fact that RPPs are estimated indirectly, that is, by solving the corresponding inverse problem, which leads to instability. Below, we consider interference-suppressing algorithms for determining the relative phase permeability functions using data from unsteady-state investigations, based on the use of "a priori" information on the form of the RPPs.

2. BASIS OF THE PARAMETRIZATION OF THE RELATIVE PHASE PERMEABILITY FUNCTIONS USING DATA FROM STEADY-STATE INVESTIGATIONS

To avoid instability when finding the functional dependences it is necessary to use regularizing algorithms [2] which ensure that the solution of the inverse problem of determining the RPPs is well posed. We know that parametrization of the required functions is one of the efficient methods of regularization. The relative phase permeability functions are therefore considered in the form [3]

$$f_1(s) = A_1\left(\frac{s - s_c}{1 - s_c}\right)^{N_1}, \quad f_2(s) = A_2\left(\frac{s_T - s}{s_T - s_C}\right)^{N_2}$$

where the parameters A_1, A_2, N_1 and N_2 are determined from the condition for the theoretical dependences to be as close as possible to the experimental ones. However, an analysis of steady-state investigations show that the form of the phase permeability curves often differs from that of a power curve. Furthermore, porous media with different physicochemical properties can be characterized by RPP function curves of a completely different type.

A well-founded parametrization of the RPP functions can be carried out if the form of the RPP functions, determined for lithologically similar samples of porous media by steady-state methods of investigation, is known from independent experiments or from the literature. This is based on the fact that the experimental dependences, obtained for different samples of porous media with similar physicochemical properties, can be represented in a certain universal form by changing to normalized coordinates, proposed for the first time by Collins

$$x = \frac{s - s_C}{s_T - s_C}, \quad y_i = \frac{f_i(s)}{F_i}$$

where F_1 and F_2 are the RPP functions of the displacing agent and the displaced fluid when $s = s_T$ and $s = s_C$, respectively. In this system of coordinates, the RPP functions measured for different (but lithologically similar) samples fall on unique universal curves, the analytic expressions for which are sought in the form $y_i = F_i(x, p)$, where $p \in R^N$ are parameters which are determined using well-known methods for establishing experimental relationships, and N is the number of these parameters.

As an example, we shall consider the RPP functions recorded while carrying out steady-state investigations on lithologically similar samples of porous media from the Priobskii deposit. Plots of these functions in normalized coordinates are shown in Fig. 1 (the open circles refer to the RPP function of a sample for which $s_c = 0.298$ and $s_T = 0.695$, while the solid circles refer to the RPP function of a sample with $s_c = 0.4$ and $s_T = 0.707$). It can be seen that the experimental points do, in fact, fall on the unique curves which can be represented in the analytic form

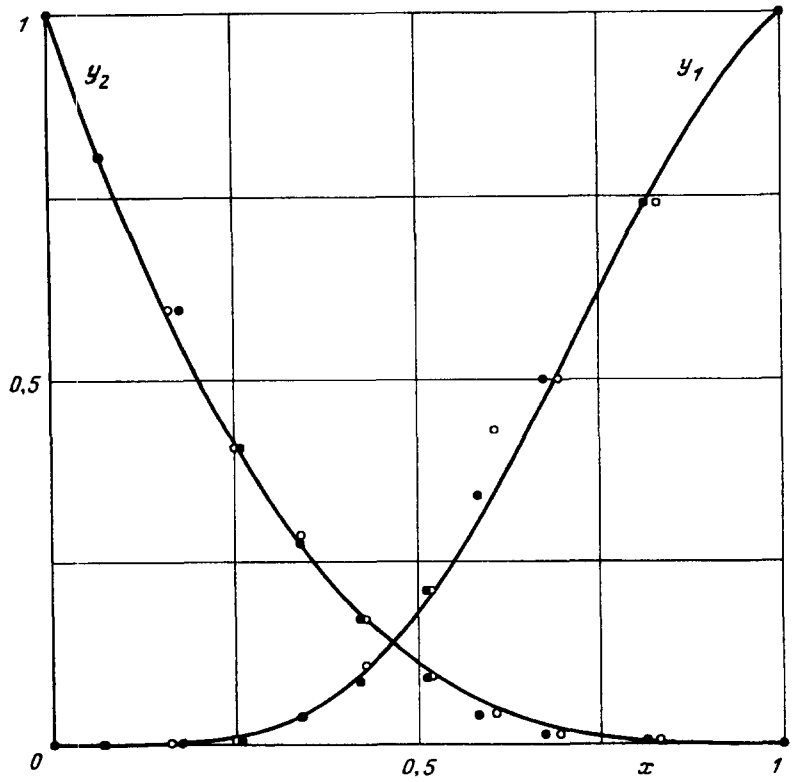

Fig. 1.

$$y_1 = x^{p_1+p_2x}, \quad y_2 = (1-x)^{p_3+p_4x}$$

As a result of the instability of the inverse problem, the problem of reducing the complexity of the model (that is, the problem of reducing the number of parameters p which are determined using data from unsteady-state investigations) is important. In order to reduce the number of required parameters, standard values, that is, the values obtained in steady-state investigations of lithologically similar samples, can be assigned to some of them. In a number of cases, relations between parameters, which follow from the data obtained in steady-state experiments, can be used to simplify the model.

Sometimes, displacement experiments are interrupted without waiting for the steady-state filtration conditions to become established. In this case the quantities s_T and F_1 are included among the unknown parameters and, in order to reduce the complexity of the model, some of the parameters p can be assigned values p^e which are determined from the data of standard experiments.

3. AN ALGORITHM FOR SOLVING THE INVERSE PROBLEM

The solution of the inverse problem reduces to minimizing the residual

$$I_0(p) = \sum_i [\alpha(\Delta P^0(\tau_i) - \Delta P(\tau_i, p))^2 + (V_2^0(\tau_i) - V_2(\tau_i, p))^2]$$

with respect to the parameters $p \in R^N$. The functions $\Delta P(\tau)$ and $V_2(\tau)$ are found using the formulae derived in Section 1, $\alpha = (V_2^*/\Delta P^*)$ is a coefficient which takes account of the difference in the scales of variation and in the dimensions of the quantities V_2 and ΔP, and V_2^* and ΔP^* are their characteristic values.

The minimum value of the residual is determined by the method of successive descent, and minimization with respect to each of the required parameters is carried out by the golden section method. In order to restrict the search region, a certain initial point p_0 (the first approximation) is chosen and the solution is sought within a small neighbourhood of this point. The point p_0 can be found, in particular,

by the Monte-Carlo method, that is, by a random selection of points p from a certain domain and by comparing the values of the residual at these points.

The optimal number of required parameters N can be found using the following two methods.

The method of structural minimization of the mean risk. The problem of the correct relation between the complexity of the model which is being identified and the amount and level of error in the available data can be solved using the method of structural minimization [4]. It is found that, if the solutions define a structure in a permissible set, then, together with minimization of the empirical risk (of the residual) within the elements of the structure, an additional possibility of minimization with respect to the elements of the structure appears. This enables one to find a solution which gives a deeper guaranteed minimum of the mean risk than a solution which yields the minimum value of the empirical risk in the whole permissible set of solutions.

In the case under consideration, the structure is specified by the RPP parametrization set. We shall confine ourselves to the treatment of the following four models, the complexity of which is determined by the number of required parameters

$$1)\ f_1(s) = F_1 x^{p_1},\quad f_2(s) = F_2 x^{p_2}$$

$$2)\ f_1(s) = F_1 x^{p_1 + p_3 x},\quad f_2(s) = F_2 x^{p_2}$$

$$3)\ f_1(s) = F_1 x^{p_1},\quad f_2(s) = F_2 x^{p_2 + p_4 x}$$

$$4)\ f_1(s) = F_1 x^{p_1 + p_3 x},\quad f_2(s) = F_2 x^{p_2 + p_4 x}$$

Estimates of the values of the parameters $\tilde{p}$ in these models are determined using the initial choice of $(\Delta P^0(\tau_i), V_2^0(\tau_i))$, $i = 1, \ldots, M$ by minimizing the residual (the empirical risk functional) $I_0(p)$: $\tilde{p} = \arg\inf I_0(p)$.

The stability of the solution of the inverse problem is ensured by selecting the relations of optimal complexity from the four models which have been presented above. It has been shown [4] that, for each N, it is possible to construct, with a probability $1 - \eta$, an upper estimate of the mean risk of the form

$$I(N) = I_0(\tilde{p})\Omega\left(\frac{N}{M},\ \frac{\ln\eta}{M}\right)$$

where the factor Ω determines the degree of correspondence between the complexity of the model (of the magnitude of N) and the volume of the sample M. As a rule, the magnitude of the first factor decreases as N increases, while the magnitude of the second factor increases. The method of regulated minimization of the mean risk involves finding the model which minimizes the estimate $I(N)$.

The estimate

$$I = \left[I_0(\tilde{p}) \Big/ \left\{ 1 - \left(\frac{N(\ln(M/N) + 1) - \ln\eta}{M} \right)^{1/2} \right\} \right]_\infty, \quad \text{where } [z]_\infty = \begin{cases} z, & z \geqslant 0 \\ \infty, & z < 0 \end{cases}$$

is used in practical calculations.

Fuzzy constraints. The optimal complexity for a model can also be found by formalizing the vague aim of "making the residual as small as possible and the model as simple as possible" using methods from the theory of fuzzy sets [5, 6]. In particular, it may require the maximization of the criterion

$$W = ((1 - \mu_n(\tilde{I}_0)N)))(1 - \mu_c(N)))^{1/2}$$

where $\mu_n(I)$ and $\mu_c(N)$ are membership functions of the "large residual" and "high complexity of the model" fuzzy sets which are defined as

$$\mu_n(I) = \begin{cases} r^{m_1}, & r \leqslant 1 \\ 1, & r > 1 \end{cases}, \quad r = \frac{I_0}{I_c};\quad \mu_C(N) = \begin{cases} (2N/M)^{m_2}, & 1 \leqslant N \leqslant M/2 \\ 1, & N > M/2 \end{cases}$$

where $\tilde{I}(N)$ is the minimum value of the residual which is obtained by varying the N parameters p,

$I_c = \tilde{I}(N_1)$, where N_1 is a certain initial number of parameters ($N_1 - 2$, for example), and m_1 and m_2 are exponents which determine the behaviour of the algorithm when there is a decrease in the residual and an increase in the complexity of the model (for instance, if $m_2 < 1$, the model is acknowledged as being complex even for small N, while, when $m_2 > 1$, the number of parameters can be slightly increased).

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